Answer all questions. Write your answers in the spaces provided.

1. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4 - a_n$$
$$a_1 = 3$$

Find the value of

(a) (i) a_2

(ii) a_{107}

(2)

(b) $\sum_{n=1}^{200} (2a_n - 1)$

(2)

2. A circle C has equation

$$x^2 + y^2 + 4x - 10y - 21 = 0$$

Find

(a) (i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The point $P(5, 4)$ lies on C.

(b) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

3. (i) Use algebra to prove that for all real values of x

$$(x - 4)^2 \geqslant 2x - 9$$

(3)

(ii) Show that the following statement is untrue.

$2^n + 1$ is a prime number for all values of n, $n \in \mathbb{N}$

(1)

4. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2-\frac{1}{4}x\right)^6$$

(4)

(b) Given that x is small, so terms in x^4 and higher powers of x may be ignored, show

$$\left(2-\frac{1}{4}x\right)^6+\left(2+\frac{1}{4}x\right)^6 = a+bx^2$$

where a and b are constants to be found.

(3)

5. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and £x is the selling price of the watch.

According to this model,

(a) find, using calculus, the maximum possible annual profit.

(6)

(b) Justify, also using calculus, that the profit you have found is a maximum.

(2)

6. $\mathrm{f}(x) = kx^3 - 15x^2 - 32x - 12$ where k is a constant

Given $(x - 3)$ is a factor of $\mathrm{f}(x)$,

(a) show that $k = 9$

(2)

(b) Using algebra and showing each step of your working, fully factorise $\mathrm{f}(x)$.

(4)

(c) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$$

giving your answers to one decimal place.

(2)

7. Kim starts working for a company.

- In year 1 her annual salary will be £16 200
- In year 10 her annual salary is predicted to be £31 500

Model A assumes that her annual salary will increase by the same amount each year.

(a) According to model A, determine Kim's annual salary in year 2.

(3)

Model B assumes that her annual salary will increase by the same percentage each year.

(b) According to model B, determine Kim's annual salary in year 2. Give your answer to the nearest £10

(3)

(c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10

(3)

8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b\log_2 3$, where a and b are constants to be found.

(4)

(ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2\log_5(y + 1) - 1$$

(5)

9. (a) Show that the equation

$$\cos\theta - 1 = 4\sin\theta\tan\theta$$

can be written in the form

$$5\cos^2\theta - \cos\theta - 4 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < \frac{\pi}{2}$

$$\cos 2x - 1 = 4\sin 2x\tan 2x$$

giving your answers, where appropriate, to 2 decimal places.

(4)

10.

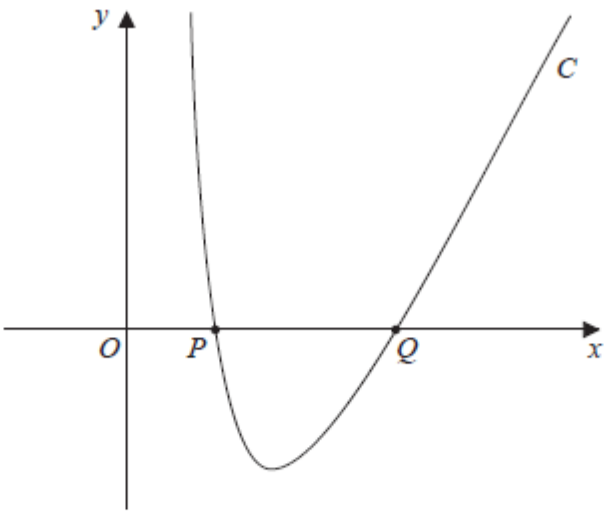

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{36}{x^2} + 2x - 13 \qquad x > 0$$

Using calculus,

(a) find the range of values of x for which $f(x)$ is increasing,

(4)

(b) show that $\int_2^9 \left(\frac{36}{x^2} + 2x - 13\right) dx = 0$

(4)

The point $P(2, 0)$ and the point $Q(6, 0)$ lie on C.

Given $\int_2^6 \left(\frac{36}{x^2} + 2x - 13\right) dx = -8$

(c) (i) state the value of $\int_6^9 \left(\frac{36}{x^2} + 2x - 13\right) dx$

(ii) find the value of the constant k such that $\int_2^6 \left(\frac{36}{x^2} + 2x + k\right) dx = 0$

(3)